

1. Use the formula for the sum of a geometric series to find a power series representation for  $f(x) = \frac{5}{1-2x}$ .
2. Our goal is to find a power series representation for  $f(x) = \frac{1+x}{1-x}$ .
  - (a) Find the power series for  $g(x) = \frac{1}{1-x}$  and its radius of convergence.
  - (b) Use part (a) to find a power series representation for  $h(x) = \frac{x}{1-x}$ .
  - (c) We want to add the two series together to get a series representation of  $f(x)$ . This will be easier if both series have terms with the same power of  $x$ . Use an index shift to rewrite the series for  $h(x)$  so that its terms include  $x^n$ .
  - (d) Now we want to add the two series together. Try writing each series in expanded form and then writing a new series expression for the sum.
  - (e) What is the radius of convergence of the final series for  $\frac{1+x}{1-x}$ ?
3. Now let's find a power series representation for  $f(x) = \frac{2x}{(1-x)^2}$ .
  - (a) Find a power series representation for  $g(x) = \frac{1}{(1-x)^2}$ .  
Hint: Use the power series for  $\frac{1}{1-x}$ .
  - (b) Use (a) to find a power series representation for  $h(x) = \frac{2x}{1-x}$ .
4. Let  $f(x) = \frac{1}{1+x^2}$ .
  - (a) Find a power series representation for  $f(x)$ , including the radius of convergence.
  - (b) Integrate to find a power series representation for  $g(x) = \arctan(x)$ . Use the initial condition  $g(0) = 0$  to solve for the value of the constant.
5. Find a power series representation for  $g(x) = \frac{x}{9+x^2}$ .  
Hint: Find a representation for  $\frac{1}{1+(x^2/9)}$  first.
6. Use the antiderivative of  $f(x) = \frac{1}{(x+3)^2}$  to find a power series representation of  $f(x)$ .
7. Use the previous problem to find a power series representation of  $g(x) = \frac{x^3}{(x+3)^2}$ .